ANGULAR APERTURE
OF
MICROSCOPE OBJECTIVES

BLACKHAM

To Frank Crisp. Esq
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with the compliments of
the Author

ON

ANGULAR APERTURE

OF OBJECTIVES

FOR THE MICROSCOPE.

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BY

GEO. E. BLACKHAM, M.D., F.R.M.S.,
President Dunkirk (N. Y.) Microscopical Society, Etc., Etc.

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EXPLANATORY NOTE.

To those readers who are familiar with mathematics and optics, it will doubtless appear strange that in the following pages no use has been made of mathematical formulæ.

There are, however, a large number of microscope users to whom a purely mathematical discussion would be either unintelligible, or uninteresting, but who are, nevertheless, desirous of information upon the much-talked-of subject of Angular Aperture.

It is hoped that by the intentional selection of untechnical methods of discussion, and the liberal use of carefully calculated and accurately drawn diagrams, this paper has been made interesting and useful to non-mathematical readers without entirely losing its value for those who would have prefered the greater conciseness of a purely mathematical discussion.

ON ANGULAR APERTURE OF

OBJECTIVES FOR THE MICROSCOPE.

IN the newspaper report of a recent popular lecture on the microscope, I found the following excellent statement of the primary function of the object-glass of a telescope, viz.: "Thus we see how this piece of glass, so shaped and polished, gathers up the otherwise diffused and lost rays of light that issue from these distant objects. It collects at least a thousand times the quantity of light that the unaided eye could seize, and brings the whole rescued bundle of rays to a focus, in which the image of the source from which they stream is brilliantly reproduced. This image the eye can get near, and by the aid of a magnifying lens examine."

I am fully aware that newspaper reports of scientific statements are not always to be relied upon; but I hope that in this case the eminent lecturer has been reported correctly, for as it stands the passage I have quoted is a most excellent statement of the principal function of the object-glass of a telescope, and by simply leaving out the word distant it is equally true of the object-glass of a microscope. Now the angular difference between the paths of the most divergent rays, which any lens can thus gather up and *bring to a focus*, is known as the angular aperture of the lens, and forms the subject of our paper this evening. It would seem reasonable to suppose that if the value of an object-glass depends upon gathering up and bringing to a focus rays which would otherwise fail to enter the eye, and thus be dispersed and lost, that the more of these rays it could

so utilize, or, in other words, the wider its angulur aperture, the better the lens; and this, indeed, is the fact, though many microscopists, and among them the lecturer whom I have quoted, do not admit it.

Light is dispersed from every point on the surface of an object in every direction up to 180°; but, unaided, the human eye, according to Dr. David Brewster, is competent to receive only a narrow pencil of 10°. In other words, it has an angular aperture of only 10°, and can utilize only about 1-324 part of the light emanating from the surface of an object, the other 323-324, or a pencil of 85° on each side, being lost to it. It is the problem of the optician to gather up and *bring to a focus* as many of these lost rays as possible. And here let me emphasize the fact that it is only those rays which are brought to one common focus which are of value, and which should be counted in measuring the angular aperture of an objective. If rays are admitted more divergent than can be brought to a common focus, and so made to contribute to the formation of the new image, then those rays are simply detrimental, and should be cut off by means of diaphragms.

Now if the statement of the lecturer whom I have quoted, viz.: "That it is the function of the objective to collect and bring to a focus rays of light from an object too divergent to be received by the unaided eye"—be correct, and my corollary from that, "that the more of these lost rays that a given glass can so collect and bring to a focus, the better the glass," be also correct, one would naturally expect to find that the improvement or evolution of the microscope was accompanied *pari passu* by an increase of the angular aperture of the objectives; and this, indeed, we find to be the case.

When, in 1824, Mr. Tulley, of London, produced the first achromatic microscope objective made in England (a single combination of three lenses acting as one), he obtained an aperture of 18°. This seems small to us now, but in reality it was a great advance, for it is nearly double the pencil which can be received by the unaided eye.

Soon after the same eminent maker improved upon this by adding another combination in front, thus making the first English *compound* objective, and again doubled the aperture, getting, with the double combination, an aperture of 38°.

In 1829, Mr. Joseph Jackson Lister, published his celebrated paper, showing how many of the difficulties which had interfered with the use of two or more combinations together could be overcome, and exhibited, in confirmation of his conclusions, an objective, of which it is recorded that it "gave a large and correct field, and transmitted a pencil of 50°." This was indeed progress; but the end was not yet. In 1837, Mr. Thomas Ross, an eminent London optician, presented a paper to the Society of Arts, detailing his discovery of the negative aberration produced by the cover glass, and the means he had devised to neutralize it by approximating the front and middle combinations of the objective. In this paper, Mr. Ross states that he has made an improved combination, of which he

says: "The focal length is ⅛ inch, having an angular aperture of 60°, with a distance of 1-25 of an inch. Later he announced that "on several occasions the enormous angle of 135° had been obtained," and unfortunately added that "135° is the largest angular pencil that can be passed through a microscope object-glass." Chas. A. Spencer, the now famous father of American microscope making, was led to a theoretical and practical examination of the validity of this statement. The supposed theoretical grounds of the assumption not having been found to sustain Mr. Ross's position, conclusive evidence of its incorrectness was speedily obtained by the construction of a 1-12 inch objective, having an angle of aperture of 146°. And in the catalogue of Ross & Co., for November, 1874, I find advertised "Ross' New Patent Object Glasses. Devised by Mr. Wenham." 1-25th aperture about 160°. Mr. Spencer claimed 178° aperture for his 1-12 in 1851, and Mr. Tolles, of Boston, now makes lenses which have air angles of infinitely near 180°, and immersion angles greater than that corresponding to this enormous air angle.* Thus we see that, as might have been expected, the record of the gradual improvement of the microscope is the record of gradually increasing angular aperture of objectives, till at length the extreme limit of possible air angle, *i.e.*, an aperture only a differential less than 180° has been reached. That these modern wide-angled lenses are better than their narrow-angled predecessors or contemporaries, is shown by the fact that many objects entirely invisible with the narrow-angled lenses are clearly defined by wide-angled lenses of less amplifying power. Among them I may mention that Band No. XIX of Nobert, 112,594 lines to the inch, (each line being only about 1-225198 of an inch wide), has been clearly resolved with my Tolles' 1-6 of (nearly) 180° air angle; and that the flagellum of *Bacterium termo* was seen by Drysdale and Dallinger with a new wide-angled ⅛ of Powell & Lealand, when the comparatively narrow-angled 1-50 of the same makers failed to reveal the existence of this tiny appendage of the pigmy of the bacteria.

It has been objected to wide-angled lenses that they possessed less "*penetrating power*," or more properly less "*depth of focus*," than narrow-angled lenses. That is to say, that the layer of an object that could be seen without change of focus, is thinner with wide than narrow-angled lenses. If this were true, it would be an argument in favor of the wide-angled lenses instead of against them. In reality, however, it does not depend upon the aperture, but is only residual spherical aberration which can be left in and distributed in a wide-angled lens, as well as in a narrow-angled one. It is, however, as I have said, only residual error at best, and the less a lens has of it the better the lens. This will, I think, be easily seen upon an inspection of the diagram (Fig. 1), showing the action of an uncorrected plano-convex lens of crown glass. The rays from the nearer surface of the object which impinge upon the peripheral portions of the lens would, if the lens were free

* Since this was written, other makers, both here and abroad, have made and are making objectives whose immersion angles are greater than that corresponding to (infinitely near) 180° air.

from spherical aberration, be brought to a focus further back than those from the further surface of the object; as it is, however, they are both brought to the same focus by reason of the spherical aberration. Such a lens has a good deal of penetrating power or depth of focus; but its definition is not satisfactory. A common bulls-eye condenser is a good sample of this kind of lens. The same holds true of all objectives possessed of penetrating power, whatever their angular aperture. The only legitimate method of obtaining depth of focus or "penetration," is by increasing the anterior conjugate focus or working distance, so that the thickness of the layer it is desired to see on each side of the true focal plane, may be relatively small. Thus a one inch objective, with an anterior focus of ·317 of an inch, will bear amplification up to 400 diameters, and at that power might properly show, with reasonable clearness, a layer of the object on each side of the true focal plane much thicker than that which a 1-5 with only ·018 of anterior focus ought to show at the same amplification. It is, perhaps, true, that by skillful management the residual spherical aberration can be so distributed that several planes of an object may be in view at once, but this is always at the sacrifice of definition, and as the better the images the more noticeable do errors resulting from this plan of overlapping several of them become—wide-angled lenses show the defects of this plan more markedly than narrow-angled lenses, whence has arisen the fallacy that narrow-angled lenses are possessed of an inherent property of "penetration," and a residual error has been lauded as a virtue.

This much as to the value of angular aperture; now for the question "What is an angular aperture?"

I have already defined it as, The angular difference between the paths of the most divergent rays which an objective can gather up and *bring to a focus.*

Let us, however, examine some of the standard authors, and see what they have to say on the subject:

Dr. W. B. Carpenter ("The Microscope," 4th Ed., 1868) says: "The angle of aperture, that is, the angle made by the most diverging rays of the pencil issuing from any point of an object that can enter the lens."

Prof. L. Beale ("How to Work with the Microscope," 4th Ed., 1870): "The angle of aperture is the angle made by two lines from opposite sides of the aperture of the object-glass with the point of focus of the lens."

Dr. H. Frey ("The Microscope and Microscopical Technology"; Cutter's Translation; 1872): "The term, angle of aperture of a lens, denotes the angle which is formed by the focus and the two terminal points of the diameter of the lens."

Dr. Wythe, of San Francisco ("Microscopists Manual," 3d Ed., 1877): "Angular Aperture.—The angle made by the diameter of the actual aperture of an objective, and the distance from its focal point."

Mr. F. H. Wenham is thus quoted and endorsed by Chas. Brooke, Prest. R.

M. S., in his annual address to the Society, in 1875: "Mr. Wenham is unquestionably right in stating that if an isosceles triangle be described, the base of which is ten times the measured diameter of the front lens, and the altitude ten times the measured distance of the focal point from the same surface, the vertical angle of that triangle will correctly represent the *maximum available* aperture."

Now, strange as it may seem to question the concurrent testimony of such distinguished authorities, I am not disposed to accept any of the definitions I have quoted, as correct. They all lack accuracy and universality of application. To illustrate.

Let me take a hemispherical lens of crown glass, whose index of refraction is 1·525, and radius of curvature is ·015 of an inch. The diameter will, of course, be ·03 of an inch, and its principal focus for parallel rays ·0286 of an inch. As the focal length is measured from the optical centre, which, in the case of a plano-convex lens, is situated on the convex surface of the lens, where it is cut by the optic axis, if we turn the plane side of the lens towards an object the working distance will of course be the focal length minus the thickness of the lens. In this case $f\,l$ = ·0286 — thickness = ·015 = working distance ·0136. This gives us for our isosceles triangle a base of ·03 of an inch, and an altitude of ·0136 of an inch, or, to follow out Mr. Wenham's plan, multiplying by 100· gives us a base of 3 inches, and an altitude of 1·36 inch; and on this scale I have drawn the diagram, the vertical angle of our triangle will be 95° 36′ (Fig. 2). If our object was placed in the principal focus, however, the posterior conjugate focus would be infinitely distant, and in order to get nearer to the conditions under which an objective is actually used, let us bring our posterior conjugate focus to 10 inches; this will lengthen our anterior focus to ·0157, nearly; our enlarged triangle will then be base 3 inches, altitude 1·57 inches, nearly, and the resulting angle 87° 22′ (Fig. 3). But, in point of fact, the spherical aberration would be so great that the outer rays of this pencil would be brought to a focus at a distance considerably less, and would not enter into the formation of the image at 10 inches, but would only serve to confuse it, and would have to be cut off by a diaphragm. If this diaphragm were placed behind the lens, the diameter of the front, and the distance to the focal point, and consequently our triangle, would remain unchanged; and our triangle would consequently indicate an angular aperture far beyond the real available angular aperture of the lens. This, too, would be the case of the lens under consideration were the point of a compound objective in which the back combinations were unable to transmit the entire pencil received by the front, or, if transmitted, to correct the aberrations sufficiently to bring all the rays received by the front to a common focus at the eye-piece. This is, in fact, the case with many objectives which have not been properly corrected; they will admit rays far more divergent than the extreme pencil which they can bring to a common focus at the eye-piece. For these objectives, if used dry, Mr. Wenham's rule will give an

aperture in excess of the maximum available aperture, thus giving undue credit to faulty lenses. It is this class of lenses which are improved by having the aperture reduced, as detailed by Dr. Piggott, but the argument is not, therefore, that all wide-angled lenses would be benefited by a reduction of aperture, but only those in which the marginal rays have not been properly corrected.

Now, in order to understand the matter fully, it is necessary for us to remember that in the vast majority of cases, objects viewed through the microscope are seen under very different conditions from those under which we ordinarily view objects with the naked eye.

In the case of objects viewed with the naked eye, we see them without the interposition between them and the eye of any medium but air, the index of refraction of which is so small as to be practically of no effect, and consequently the rays of light which radiate from them, either primarily or by reflection, reach the eye without appreciable refraction, and from a distance of not less than about eight or ten inches.

Objects viewed through the microscope, on the other hand, are frequently immersed in some highly refractive medium, such as water, glycerine, or Canada balsam, are generally covered with a thin plate of glass, with parallel surfaces, and being more or less transparent are viewed by rays of light which pass through them from below, and so through the cover glass to the microscope and to the eye.

The importance of this distinction will be seen when we come to discuss the question of immersion apertures, and especially apertures beyond the extreme limit possible for dry objectives.

Let us now take our hemispherical lens of crown glass, as before, and suppose it to be the front lens of an objective whose posterior combinations are so arranged as to leave its anterior focal length unchanged while correcting its aberrations, and thus bringing all the rays that can enter it into a common focus at the eye-piece. Allow ·002 of an inch for the setting; we should still have ·028 of an inch available front; our enlarged triangle will then have for dimensions: base 2·80, altitude 1·57, vertical angle 83° 26′, nearly, (Fig. 4). This, then, would be the angular aperture of that objective under those circumstances.

Of course, if the radiant point were placed closer to the lens than its principal focus, the angle made by the extreme rays which entered the face of the glass would be increased, but in that case they would be still somewhat divergent on emerging from the posterior side of the front lens, and without further refraction would not meet at any posterior conjugate focus, and of course form no image. This further refraction is afforded by the posterior combinations of the compound objective, and as a rule the actual focal length of a compound objective is much less than that of its front lens taken alone; and this focal length of the objective, as a whole, varies with the distance of the posterior conjugate focus at the eye-piece; that is to say, with the length of the tube of the microscope, and also with

the distance between the front lens and the middle combination, or lens, of the objective—a distance which is variable in objectives having correction for thickness of covering glass.

It is quite possible, however, that the posterior combinations might not be capable of transmitting the most divergent rays which could pass through the front, or even if capable of transmitting them might not be capable of correcting their aberrations (which are always the greatest for marginal rays). In either case the rule propounded by Mr. Wenham, and endorsed by President Brooke, to which I have before referred, would give inaccurate results, and the vertical angle of an isosceles triangle whose base was ten times the measured diameter of the front lens, and the altitude ten times the distance of the focal point, would *not* correctly represent the maximum available aperture, but something in excess thereof. In fact, I think it is demonstrable that this rule can never give the maximum available aperture of an objective.

Mr. Wenham, himself, seems to have become conscious of this, for he has since proposed other methods and modifications, each in turn announced to be the only reliable one, till the student who has tried to follow him through his discussion of this matter gets lost among his numerous amendments. One method of his, set forth in the London *Monthly Microscopical Journal* for March, 1874, is to place in the focus of the microscope a slide, the upper surface of which is covered with some opaque material (he suggests platinum foil here), through which a slit is cut, the edges of which serve to cut off extraneous rays, and then take the aperture in the usual way with a sector; that is, by placing a light in front, and either rotating the microscope around the object as centre (as can be done with Beck's, Zentmayer's and Tolles' largest stands), till the light disappears from the centre of the field, or by making the light traverse the circumference of a circle of which the object is the centre (as can be done with the stand made for me by Mr. Tolles) till the same result occurs. This will give one-half the angle, and of course multiplying by two will give the entire angular aperture. In this paper (March, 1874) Mr. Wenham says, "it is preferable to open the slit till the edges appear in the margin of the field." He changed his mind about this afterwards, and stated, "the narrower the slit the more accurate the result will be." I can not give date and page for this, but he quotes it himself in the *Monthly Microscopical Journal* for December, 1876, and adds: "This means strictly that for absolute accuracy we must approach to a line and cut off all rays in the focal plane on either side, quite up to the axis of the object-glass."

I may here say that this plan has some of the faults of his old triangle method; it will give the most oblique ray that can enter the lens from the object, but it will not give a clear indication whether such rays can be utilized to produce a *well-defined* image of the object. I know of object-glasses that give by this method a very large angle, but from lack of accurate correction for

the very oblique rays, their effective angle is much less than the one indicated by this plan.

This difficulty also occurred to Mr. Wenham, and he moved another amendment on himself, so he gives the diagram which I have copied here (Fig. 5), and the following description: "I now adopt the following method of measuring apertures: *a* is the working diameter of an object-glass; *b* the central pencil or true angle of aperture; *c c*, oblique or lateral pencils enclosing the field of view; *d d*, a slit of considerable width, with parallel edges attached to a glass slip, *e*. In order to measure apertures, the object-glass is first adjusted and focused on the upper surface of the glass slip. One edge of the slit is now brought forward so as to exactly bisect the field of view, half of which will appear quite dark. Over the eye-piece is now placed a cap containing a biconcave lens of about half an inch radii; by means of this and the movement of the sliding containing tube, a distinct telescopic image of a distant lamp, or other bright object, may be obtained through the open half of the object-glass. Turn the open end *away* from the lamp by rotating the microscope, and the flame will suddenly disappear at the point where it is observed by the edge of the slit. Mark this as zero! Now remove the lens from over the eye-piece, bring back the slit till the opposite edge obscures the other half of the field, and again exactly bisects it, seeing that the plane, *e*, is still in focus; replace the cap and turn the microscope till the flame again vanishes, and true aperture will be indicated."

In the last issue of the *Monthly Microscopical Journal* (Nov. and Dec., 1877), Mr. Wenham has changed his plan again; he has abandoned his slit in the focus of the objective, but still uses the microscope as a telescope. He says: "The arrangement that I now make use of consists of an 'examining lens' placed over the lowest eye-piece. This lens is a plano-convex achromatic of near four-tenths of an inch focus, contained in a tube sliding in an outer one, firmly fitting on to the eye-piece nozzle; at a distance of one and a half inches behind the lens there is a removable cap, containing a thin plate with a central stop (he means a *hole*) of one-fiftieth of an inch in diameter. The small size of this stop, and the distance that it is placed from the lens, ensures the fixed direction of the eye in the axis, and prevents any rays, except those of the central pencil, from entering. By means of the draw-tube, a lamp flame, or other object taken for an index is focused for distinct vision without the stop. Replace this and take the angle of the objective, either by rotating on a sector in the usual way, or by measuring the angle between two objects set the requisite distances asunder, the apex being at the focal point of the object-glass."

To these last two methods it might be very well objected that most of the conditions under which a microscope is used in practice are here reversed; that distant objects are viewed instead of near ones; diminished images seen instead of enlarged ones; that no account is made of the effects of cover glass or cover

correction, and that it is manifestly absurd and unscientific to use a microscope as a telescope in order to determine its qualities (or one of them—angular aperture) when used as a microscope; but instead, we will let Mr. Wenham answer himself. In the *Monthly Microscopical Journal*, for November, 1872, Mr. Wenham says: "Prof. Govin has proposed to associate the measurement of the angle of aperture with the simultaneous view of an object distinctly defined, like the flames of two candles placed asunder, or two white strips separated on a black screen, to the limit of distinct visibility; the angle from these two points to the focus of the object-glass will represent the aperture. The microscope is thus converted into a kind of telescope by means of a pair of lenses over the eye-piece, similar to what is known as Ross's 'examining glass.' Unfortunately for the success of this plan, different optical combinations at the eye-piece give different results by elongating or shortening the conjugate focus." In view of this statement of Mr. Wenham himself, of the effect of different optical combinations at the eye-piece giving different results, the consistency of his proposition in 1876 to use a "*biconcave* lens of about half an inch radii" over the eye-piece, and in 1877 to use "a plano *convex* lens of near four-tenths of an inch focus" over the eye-piece, to obtain *the same result*, becomes beautifully apparent.

My only apology for taking up so much time in quoting the various conflicting dogmas promulgated by Mr. Wenham in regard to the measurement of angular aperture, is that he has been the most prominent participator on one side of this discussion, which has been carried on for several years; and as he is still quoted by his admirers as an authority on the subject, it seemed best to quote liberally from his contributions to the confusion of knowledge on this subject, in order to show, by his own writings, the condition of self-contradiction and general absurdity to which he has been reduced by his successive attempts to overthrow Mr. Tolles' claim to having constructed objectives of extreme angle; and to thereby demonstrate the total untrustworthiness of Mr. Wenham as an authority on angular aperture, however eminent he may be as an inventor of binocular arrangements, reflex illuminators, and "patent" objectives.

The question then arises, "How should angle of aperture be measured?" and the answer is that the angle of aperture being the angular distance between the extreme rays of the widest pencil which the objective can gather into one common focus, with the production of a *well-defined* image at the eye-piece, it is necessary to measure the angle of the objective *when in actual use* on the microscope, with an object in the centre of the field, and giving, in conjunction with the eye-piece, the most perfect definition possible. If now we can measure the angle contained between the most oblique ray of the pencil actually utilized in the production of a well-defined image, and the optical axis of the instrument, it will be just one-half the available aperture of the lens; and by simply doubling it we will, of course,

obtain the full available aperture.* We will first consider the case of an objective when used on an object uncovered in air.

Objects, when examined in the microscope, are usually placed upon slides of crown glass, whose surfaces are parallel to each other and at right angles to the optic axis of the microscope.

I have such a one upon the stage of my microscope, and Fig. 6 represents the same on an enlarged scale. The light enters from below, is refracted toward the axis in the body of the slide, and the surfaces of the slide being parallel, and the ray being incident from and emergent into the same medium (air), it follows, from a well-known optical law, that the angle formed by it with the normal is the same on both sides of the slide; but the optic axis of the microscope is in this case the normal, whence it follows that if we can measure the angle of incidence below the slide, or in other words, the angle of illumination, we shall obtain the angle of emergence, which, when multiplied by two, will give us the angle of the lens.

I have here a Student ¼, sent me by Mr. Tolles, and stated by him to have aperture of about 110°. The diameter of the exposed surface of its front lens is 0·16 of an inch, and its working distance, when used uncovered with the ½ in. solid eye-piece on this stand is ·015, which gives for the vertical angle of Mr. Wenham's triangle 158° 46′, nearly, (Fig. 7). If, however, we take the diameter of the light spot seen when looking through the front of the lens, that is the clear aperture, we will find it to be ·077, which, with the same frontal distance, will give us about 137° 26′ for our vertical angle (Fig. 8). Each of these is so largely in excess of the angle claimed by the maker as to suggest a doubt of the correctness of either, and we will proceed to actual measurement. Attached to my stand is a graduated circle, whose centre is in the horizontal plane of the object; on this circle rides a fitting carrying mirrors and accessory holder, and an index which stands at zero when the centre of the fitting is in the optic axis of the instrument. Removing the accessory holder, we use its socket as a candlestick, to hold a toy candle, and turn the microscope so that its body is horizontal, and get from our tiny candle flame central illumination. Now, turning the holder in the graduated circle, we swing the light around our object as a centre till either the image becomes imperfect or the centre of the field darkened, and we get one-half the useful aperture of the lens in this condition, which is 50°, making the total aperture at uncovered, 100°. (Fig. 9).

There is one objection to this plan, which is that, on account of the refraction at the lower surface, the ray does not proceed directly from the lamp to object. This objection is valid, but resulting error is very small, on account of the small

* In practice it is much better to take two readings—one right and one left of the optic axis of the microscope—and add them together to obtain total angle of aperture, as by this method any errors arising from lack of absolute accuracy in centering the illumination are eliminated.

thickness of the slide; but even this slight error will be eliminated by the method which I shall finally propose.

Let us now take the case of an object mounted dry under a glass cover. I have such a one here. A slide is rendered opaque by a coating of photographers' collodion, which has been blackened by exposure to light. In this a narrow slit, 1-300 wide, has been cut with a needle point, and diatoms have been mounted on the slide, so that some of them will lie in the slit, and one portion flooded with balsam, and the rest left dry, and all covered with a disc of glass just ·009 inch thick. We now adjust our quarter, with same eye-piece, over one of the diatoms on the dry part, finding it necessary to close the combinations very considerably from their adjustment for uncovered. This does not change the diameter of our front, but it does change the angular aperture of the lens. We now have a working distance of ·009, which, plus thickness of cover glass, ·009, and air space, ·001, gives frontal distance of ·019. Taking the diameter of light spot of our lens for a base, and ·019 for vertical light of our isosceles triangle, Mr. Wenham's rule gives us 127° 28′ (Fig. 10) for angle; if we take the exposed diameter of our front lens for base the angle is 153° 16′. (Fig. 11). Measuring once again by swinging our candle around the object, we find that good definition ceases at about 55° from axis, giving 110° as available angle of our lens when adjusted for cover 9-1000 inch thick. This is more than the angle when adjusted for uncovered, though the frontal distance is larger, the increase being due to the change in the relative positions of the combinations of the objective itself.

Tracing now our ray from the candle to objective (Fig. 12), we find it incident from air to lower surface of slide at 55°, refracted in the glass to 32° 30′, emergent into air below the cover at 55°, at which it is incident to lower surface of cover; refracted in cover to 32° 30′, and finally emergent into air at 55°. It is impossible to show graphically the deviation of the ray in the tiny air space of 1-1000 of an inch in thickness, without drawing diagrams on an enormous and unwieldly scale—this tiny air space is an important factor, however, as we shall hereafter see.

I have used the slit here, not because it is of any real importance, but to approximate my method to one of Mr. Wenham's, and because it affords a convenient method of ascertaining when the object is in the centre of the field. If our object is immersed in balsam, and covered, the result will be practically unchanged, except that the ray of light will suffer no appreciable refraction between the object and cover, but will pass in a straight line through the slide, balsam, and cover, and have its final emergence into air, the same as its incidence, below the slide. In the remainder of this paper I shall treat of the object considered as mounted in balsam and covered with a cover which, together with the balsam, shall be ·010 inch thick, unless where something different is specially stated.

It is evident that in order to increase the angle of aperture of our dry

lens, we must either increase the diameter of that portion of the front lens, which is actually used, or must shorten the frontal distance, and in either case the posterior combination of the objective must be correspondingly modified so as to transmit and bring to a common focus all the rays of the wider pencil thus admitted by the front. In practice it will be found that the increase of angle in dry lenses is obtained by shortening the frontal distance so that, with dry lenses of very wide angle, say from 150° up to nearly 180°, the covering-glass must be very thin, and the front of the objective almost in contact with the cover. The difficulty and inconvenience resulting are well known, and have done much to create the prejudice which exists in the minds of many of our older microscopists against wide-angled objectives.

Let us now consider again the case of an object immersed in balsam which has been softened with turpentine till its index is precisely the same as that of crown glass, 1·525, and covered with a thin glass 1-100 inch thick. When it is strongly illuminated, rays will proceed from it in every direction. The rays will proceed through the balsam and cover-glass, without refraction, to the upper side of the cover, whence those which reach the cover at right angles to its upper surface will proceed, still without refraction, while the oblique rays will be refracted and rendered more oblique (Fig. 13). If the emergence is into air, the ray at 40° will be so much refracted as to emerge nearly parallel to the surface of the glass, while those of 41° and more will have no emergence at all, but will be totally reflected. It is evident, then, even for a lens of nearly 180°, if dry, a balsam or glass angle of 82° is beyond the utmost limit, for even if the front of the lens comes as close to the cover as possible, without absolute contact, and so takes in rays of the greatest possible divergence in air, there will still be rays beyond which, having no emergence into air, can not reach the lens at all. Suppose, however, that the thin film of air between the cover-glass and the front of lens is replaced by one of water, or, still better, of glycerine. What then will happen? Why, the rays which suffered so much refraction upon their emergence from glass into air, will suffer much less on emergence from glass into glycerine. The ray at 40°, which was refracted to 78° 29′—on emergence into air is now only refracted to 41° 40′—and the rays between 41° and 75°, which were totally reflected at the upper surface of cover, have now an emergence into glycerine, and part of them, at least, become available (Fig. 14).

I now have here a duplex 1-6, made by Mr. R. B. Tolles, and marked Balsam Angle 95°, Air Angle 180°—the identical label which so excited Mr. Wenham's horror and amazement. It is an immersion lens, and works well when immersed in glycerine. We will use it over the same slit slide that we tested the 1-5 on. We find that over this comparatively thick cover we still have full ·003 working distance, which is ample. Our candle being in place again we turn it around the object as a centre till we find the field darkened, and on looking at the index

we find the angle reads 78°, a total air angle of 156°. A moment's inspection, however, serves to show that the field is because the light from the candle does not reach the object, which is eclipsed by the shadow of the stage. It is evident, then, that we can not determine the total air angle of this objective by this method on this stage, though it is much thinner and admits much more oblique rays than most ordinary stages.

We know, however, that the angle within the glass slide is much less than that in air, and always bears a simple ratio to, or, more accurately, that the sines of the angles made with the normal, by a ray passing obliquely from glass to air, or vice versa, bear a constant ratio to each other. If, then, we can cancel the effect of the lower surface of the slide, and let the ray pass into the slide without refraction at the lower side, we can measure the glass angle, and from that obtain, by a simple application of the law of sines, the corresponding air angle, unless the glass angle should be larger than that corresponding to air angle of 90°, in which case that would be indicated.

For the purpose of obtaining the glass angle of this lens by cancelling the effect of the lower surface of the slide, and suffering the light to pass into the slide without refraction, I shall make use of a modification of an ingenious piece of apparatus devised by Mr. Tolles, and described by him in the *Monthly Microscopical Journal* for July, 1871, and which Mr. Wenham first called a "wretched adaptation," and afterwards adopted (see *Monthly Microscopical Journal*, March, 1874, page 117). It consists simply of a plano-convex lens of such thickness that when the plane side of the lens is connected with the under surface of the slide by water, glycerine, or balsam, the thickness of lens, balsam, and slide shall equal the radius of curvature. The object on the upper surface of the slide can then be placed in the centre of curvature of the lens, and any ray reaching it from the curved surface must pass in the direction of a radius of curvature, and consequently be normal to the curved surface at the point of entrance, and consequently again can undergo no refraction there, but must pass on to the object in a straight line from the source of light. If, now, the angle that this ray makes with the optic axis of the instrument be measured, we get the angle of deviation in glass, from which we can calculate the corresponding angle in air, if the glass angle be 41° or less for the semi aperture, a glass aperture of 82°, or very nearly that, being equal to 180°, or infinitely near that in air.

In this case my hemispherical lens is of crown glass; index of refraction (mean) 1·525; radius of curvature 0·45 inch; thickness 0·33, leaving 0·12 for thickness of slide and immersion connection.

We will make the connection with a drop of soft balsam, the index of which is very closely the same as that of the glass, and mounting our candle as before, swing it round till we find the field obscured. It does not get to 78° now, but stops

at 50° (Fig. 15), indicating a glass angle of 100° for the lens;* but as less than 82° of glass angle is equal to infinitely near 180° in air, I have demonstrated that the lens has an air angle of 180°, or infinitely near that, and admits rays which could by no possibility enter a dry lens, as they would, if the object were mounted in balsam, be totally reflected at the upper surface of the cover; or, if it was a dry mount, at the upper surface of the slide. To prove this it is only necessary to move into the field that part of the slit where the balsam stops and the dry mount begins. The balsam mounted part is brilliantly illuminated, but the dry part is in darkness (Fig. 16) till the light is turned back to about 40° from axis, when the ray, being within the critical angle from glass to air, passes through, and the dry part of the slit becomes illuminated. We have here, then, two objectives; a dry ¼ and an immersion 1-6.

The most oblique ray which the dry lens can utilize, makes, in passing through the cover-glass, an angle of 32° 30′ with the optic axis of the objective, and has an emergence into air at 55° from said axis, thus giving this objective 65° of glass angle, or 110° of air angle.

The most oblique ray which the immersion 1-6 can utilize, makes, in passing through the cover-glass, an angle of 50° with the optic axis of the objective; and on account of its great obliquity, can have no emergence into air, but emerges into glycerine at 52° 23′—thus giving this objective a glass angle of 100°, a glycerine angle of 104° 46′, and an air angle of (infinitely near) 180°. That is to say, this lens can take up and utilize every ray which, radiating from a balsam mounted object, could possibly have emergence into air; and can also receive and utilize, when immersed in glycerine, a goodly pencil of rays which could never have emergence into air at all. (Fig. 17).

Will any man in his senses venture to say that the dry lens has the larger aperture of the two? I think not; and yet that is just the result to which we must come if we take the isosceles triangle method of measurement devised by Mr. Wenham, and endorsed by President Brooke. The following are the elements:

For the dry ¼—

Clear aperture of front, - - -	·077
Distance of focal point, - -	·019
Vertical angle, - - - -	127° 28′

For the immersion 1-6—

Clear aperture of front, - - -	·031
Distance of focal point, - -	·013
Vertical angle, - - - -	100°

or, 27° 28′ less than that of the dry ¼.

Figure 18 shows the two triangles superimposed, that of the dry lens being

* The glass (or balsam) angle of this lens, when adjusted for best definition over Möller's balsam-mounted probe-platte, is 95°. The thicker cover used in this experiment necessitating considerable closing of the combinations, and (in this lens) increase of aperture.

in dotted lines; and I ask, in all seriousness, if anything further is needed to demonstrate the utter absurdity of the statement that, "If an isosceles triangle be described, the base of which is ten times the measured diameter of the front lens, and the altitude ten times the measured distance of the focal point from the same surface, the vertical angle of that triangle will correctly represent the maximum available aperture."

I think, then, that I have demonstrated that: The angular aperture of an objective is the angular difference between the most oblique rays radiating from an object which the lens can gather up and *bring to a common aplanatic focus.* That, as the obliquity of these rays will differ in the same objective, according as it receives them from air, water, glycerine, or balsam, and there is but one part of the course common to all cases, and that is in the glass cover or slide, that this is the angle which should be determined by measurement, and the others calculated from it.*

I also believe that I have demonstrated that a lens may have an air angle of (infinitely near) 180°, and a glass angle still wider than that corresponding to infinitely near 180° air.

But here the old, old question of *cui bono*, What is the good of this enormous aperture? may fairly come up; and I reply, that it being the function of a microscope objective "to gather up otherwise diffused and lost rays of light that issue from an object, and bring the whole rescued bundle of rays to a focus, in which the image of the source from which they stream is brilliantly reproduced," then it may fairly be inferred that the objective which can gather up and bring to a focus the most of these otherwise lost rays, theoretically at least, is the best objective.

And here theory and practice go hand in hand. It is a task of immense difficulty to construct an objective which will gather up and bring to a common focus these extremely wide pencils, but when it is done by the hand of a master the result is splendid.

Minute details of structure, invisible with lenses of equal or greater amplifying power but smaller aperture, are clearly revealed. The images are at once sharper, clearer, and brighter. So much so that they will bear examination with extremely deep eye-pieces, and actually more amplification and better definition can be obtained with a 1-6 of 180°, than with a 1-16, 1-25, or 1-50 of say 140°. These lenses are then economical. The owner of a 1-6 of 180° (if as thoroughly corrected as this one) has no need to purchase a 1-16 or 1-25; by a change of eye-piece he can get all the amplification, definition, and resolution, that the shorter focus objectives would give him, with larger field and longer working distance. I have compared this 1-6 with a splendid 1-50. The work of the 1-6 was unquestionably superior, and with it I can work through covers 1-100 of an inch thick; while for the 1-50 extra thin covers had to be specially imported.

* This is practically the idea advanced by Prof. Abbe, of Jena, in his system of numerical apertures, though I had never heard of his plan when this paper was written.

NOTE.

In making measurements of aperture by the method here proposed, it is necessary that the experiments be conducted in a dark room, where the toy candle on the microscope is the only source of light. If other sources of light are present, they are sure to confuse the measurement, not only by the introduction of stray rays into the objective, but also by their effect upon the retina, preventing the recognition of perfect definition of the object when illuminated by the feeble light of a toy candle.

APPENDIX.

As the index of refraction of any medium differs for different parts of the spectrum, and the index for the same part of the spectrum will be found to vary somewhat in different specimens of the same medium, I have thought it best to give, as an appendix, the several indices of refraction made use of in this paper.

It has not seemed desirable to carry these indices beyond three places of decimals in any case, and thus it happens that the mean index of Air, which is usually given as 1·000294, is taken as unity (1·000). The index of crown glass (1·525), is lower than that usually given in works on Optics, but is that of the crown glass actually used by Mr. Tolles, in his objectives. The relative indices I have calculated myself from these data.

Of course all the indices given are only "means," and are for the deviation of the "Green Ray," (Frauenhofer's line E).

TABLE OF POSITIVE AND RELATIVE INDICES OF REFRACTION.

Means for Green Ray—Frauenhofer's Line E.

Positive Indices.

Air	1·000
Water	1·336
Glycerine	1·475
Balsam (thinned with Turpentine)	1·525
Crown Glass	1·525

Relative Indices.

Water to Air	0·749
" " Crown Glass or Balsam	1·142
" " Glycerine	1·102
Crown Glass or Balsam to Air	0·656
" " " Water	0·876
" " " Glycerine	0·967
Glycerine to Air	0·678
" Water	0·958
" Crown Glass or Balsam	1·034

The Positive Index for Air being taken as unity, the Relative Indices from Air to the other media will, of course, correspond with their Positive Indices as above.

Figure 1.

Spherical Aberration.
Depth of Focus or Penetration.
Not Drawn to Scale.

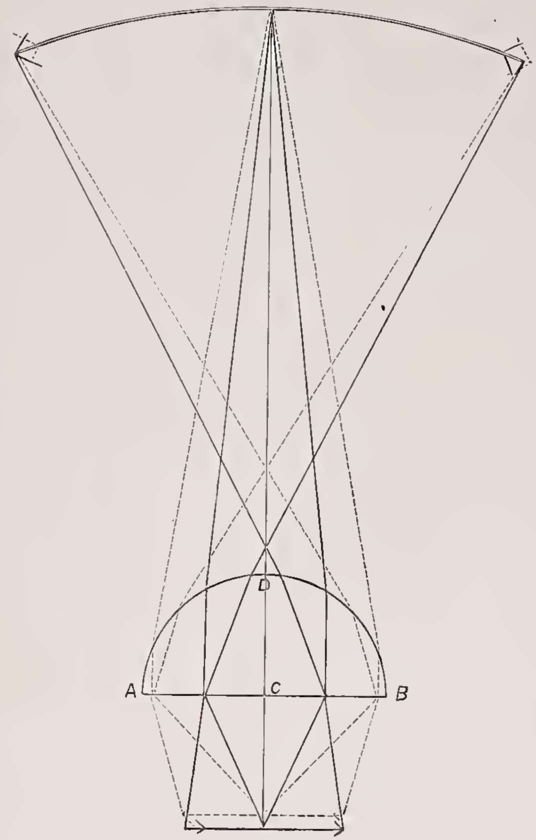

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Figure 2.

Scale × 100.

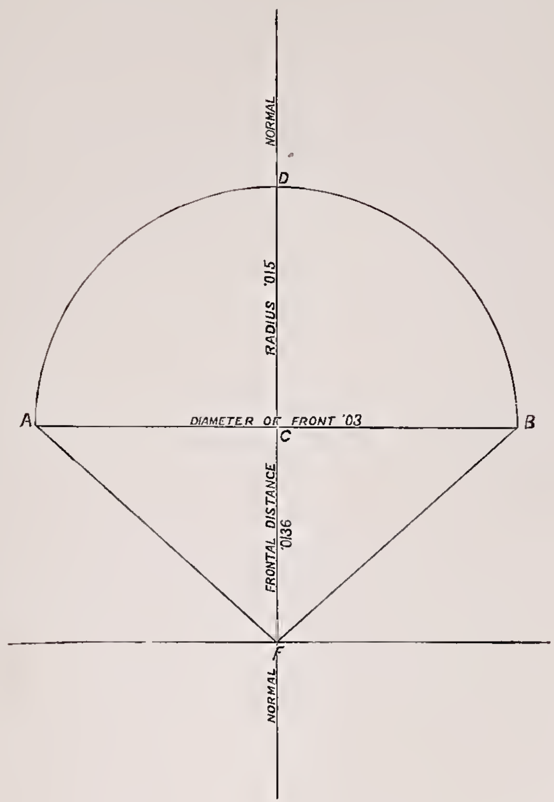

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FIGURE 3.

Scale × 100.

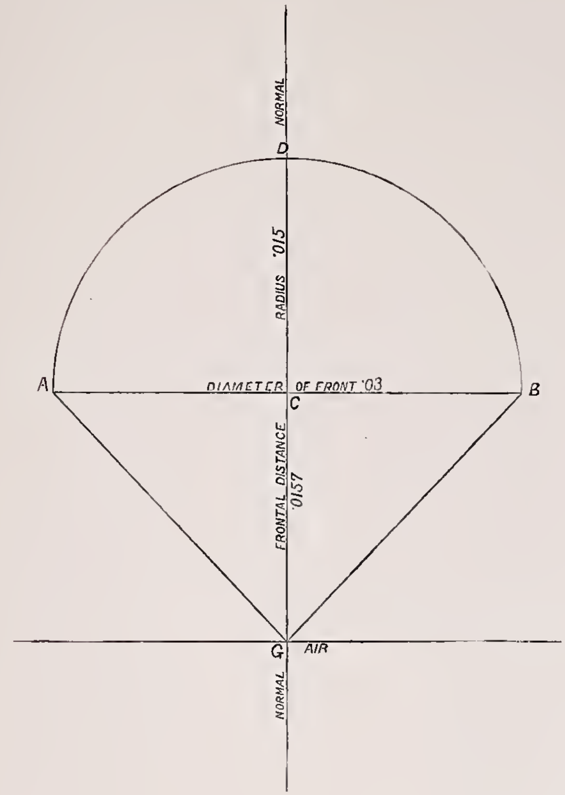

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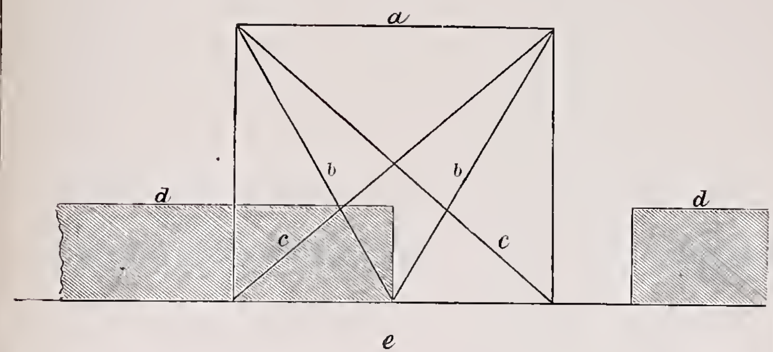
FIGURE 5.

MR. WENHAM'S APPARATUS FOR EXCLUDING ALL BUT THE CENTRAL PENCIL.

Figure enlarged two diameters from M. M. J., Dec., 1876.



a. Working diameter of Object-glass.

b. Central pencil.

c. c. Oblique or lateral pencils enclosing the field of view.

d. d. Slit of considerable width attached to,

e. Glass slip.

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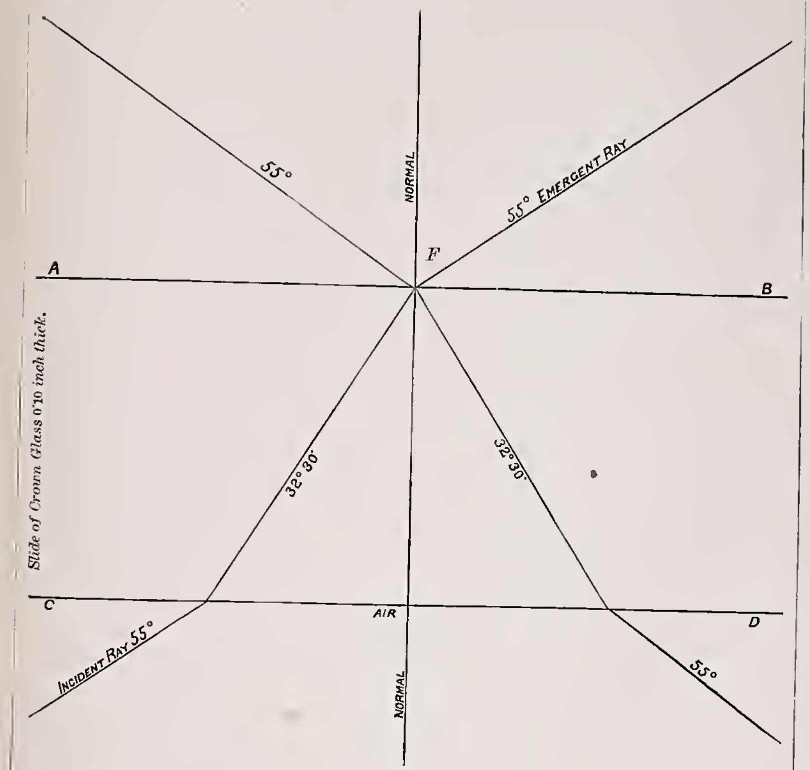
Figure 6.

Refraction in the Slide.

Scale $\times$ 20.

A. B. C. D. Crown Glass Slide 0·1 inch thick.

F. Object or Focal Point.



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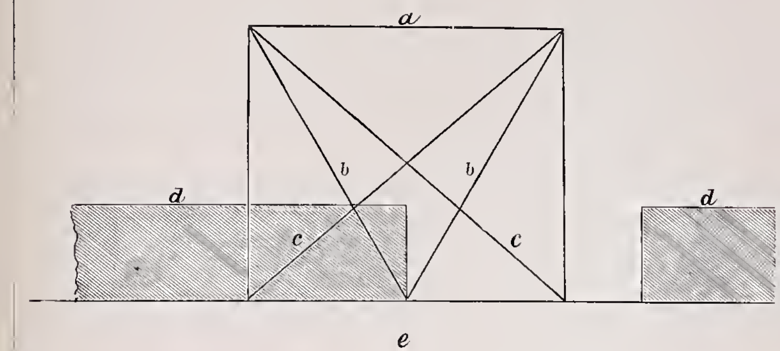
MR. WENHAM'S APPARATUS FOR EXCLUDING ALL BUT THE CENTRAL PENCIL.

Figure enlarged two diameters from M. M. J., Dec., 1876.



a. Working diameter of Object-glass.

b. Central pencil.

c. c. Oblique or lateral pencils enclosing the field of view.

d. d. Slit of considerable width attached to,

e. Glass slip.

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FIGURE 8.

Scale × 20.

Tolles' Student ¼, Dry, Uncovered Object.

C. D. Diameter of Clear Aperture of Front, ·077.
Frontal Distance Uncovered, ·015.

Resulting Angular Aperture (*a la* Wenham), 137° 26′.

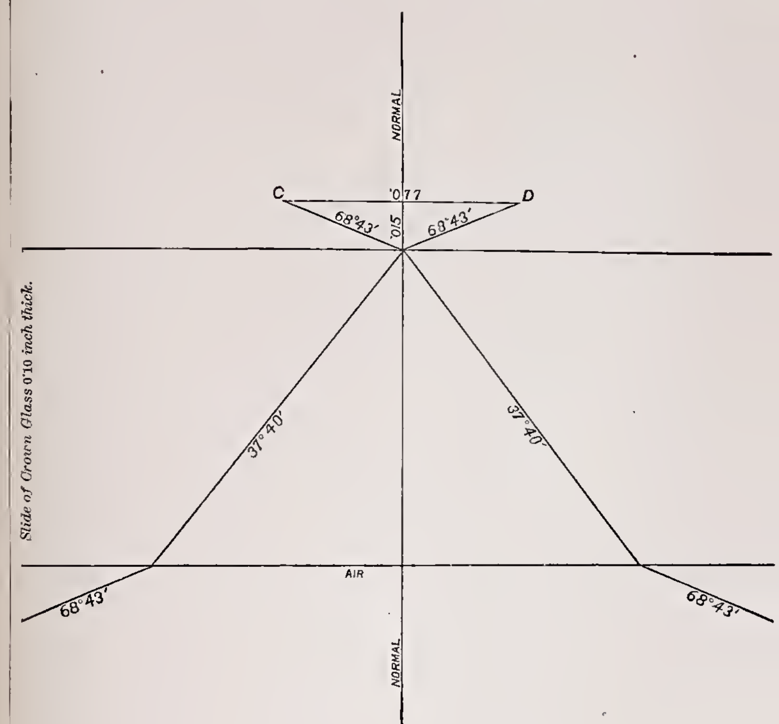

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FIGURE. 9.

Scale × 20.

Tolles' Student ¼, Dry, Uncovered Object.

Angle of Aperture, actual, by measurement, 100°.

Frontal Distance,	·015.
A. B. Resulting Diameter of Front actually utilised,	·0358.

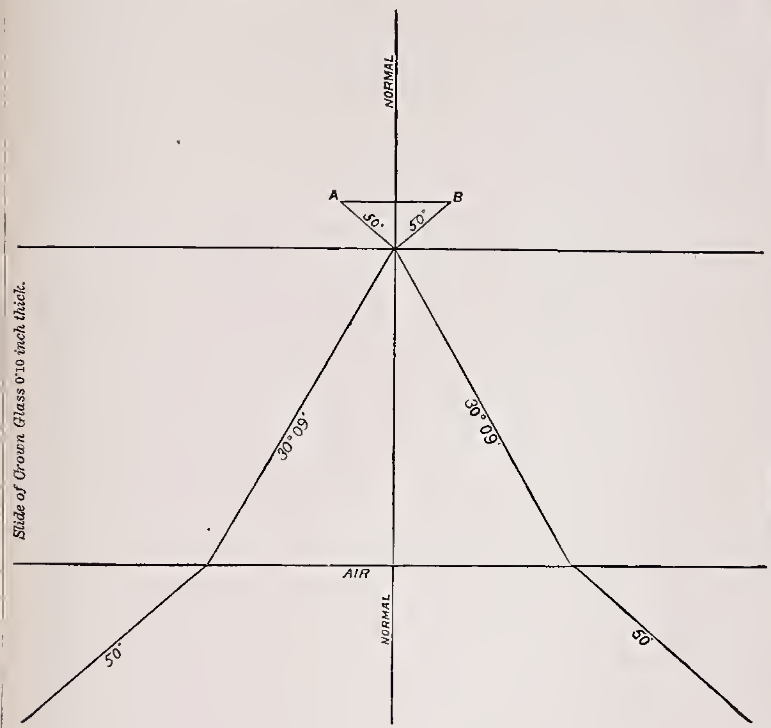

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FIGURE 10.

Scale × 20.

Tolles' Student ¼, Dry, Covered Dry Mount.

Clear Aperture of Front,	·077.
Working Distance,	·009.
Thickness of Cover and Air Space,	·010.
Total Frontal Distance,	·019.

Resulting Angular Aperture (*a la* Wenham), 127° 28′.

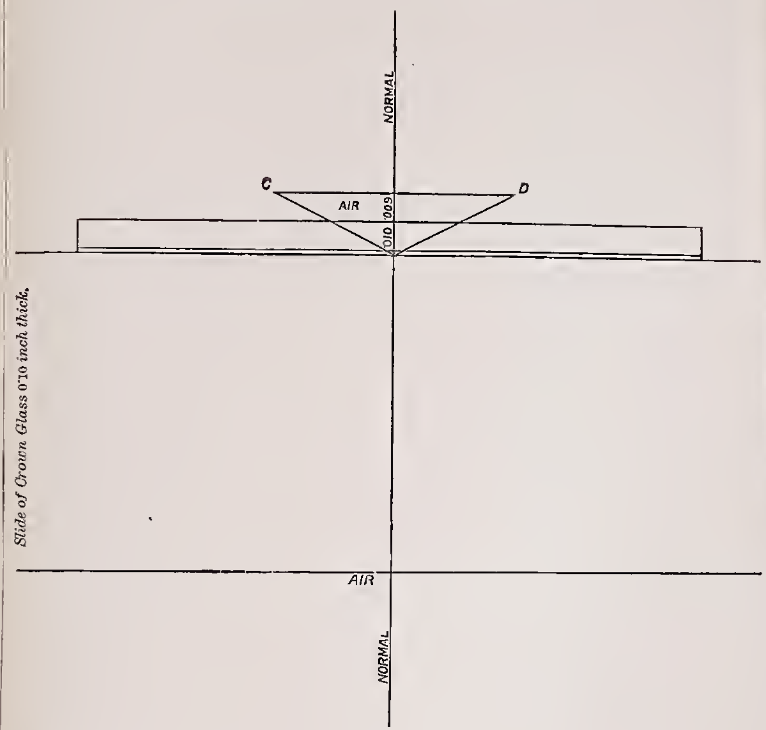

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FIGURE 11.

Tolles' Student ¼, Dry, Covered Dry Mount.

Diameter of Exposed Front,	0·16.
Working Distance,	0·009.
Thickness of Cover and Air Space,	0·010.
Total Frontal Distance,	0·019.

Resulting Angular Aperture (*a la* Wenham), 153° 16.

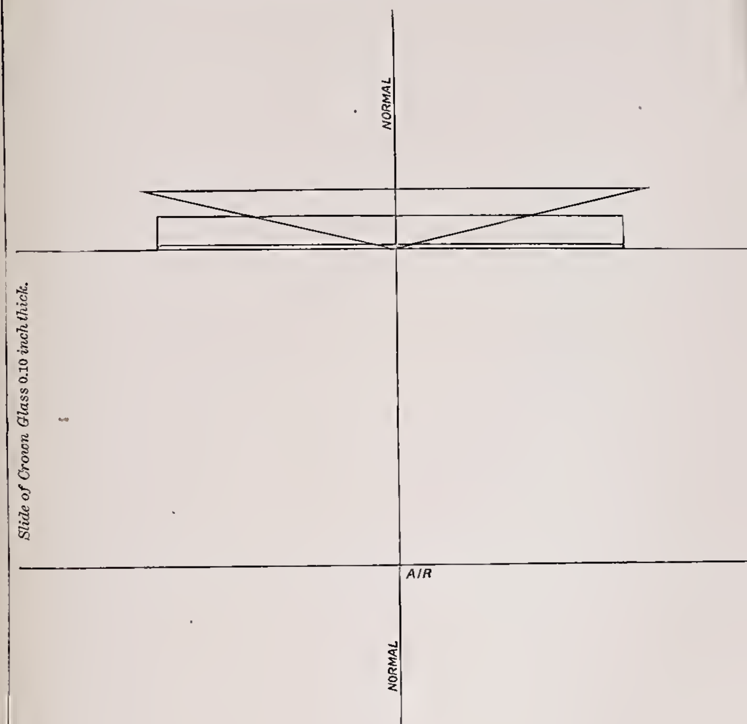

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FIGURE 12.

Scale × 20.

Tolles' Student ¼, Dry, Covered Dry Mount.

Working Distance,	·009 inch.
Thickness of Cover and Air Space,	·010 "
Total Frontal Distance,	·019 "

Air Angle (by measurement), 110°.
Glass " " " 65°.

Resulting Diameter of Front actually utilised, ·0384 inches.

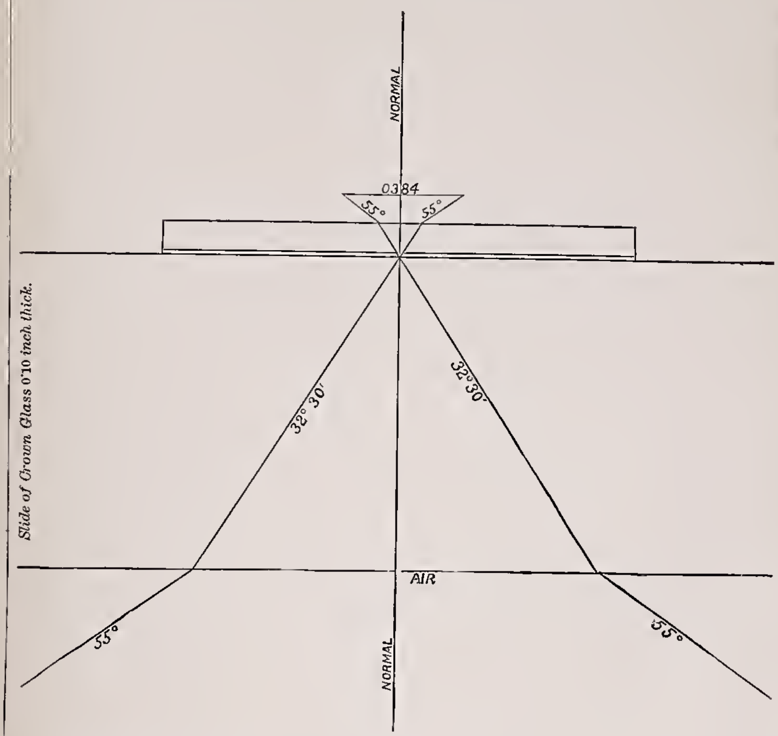

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FIGURE 13.

Balsam Mount.

Refraction of Rays passing from Crown Glass, index 1·525.
To Air, " 1·000.

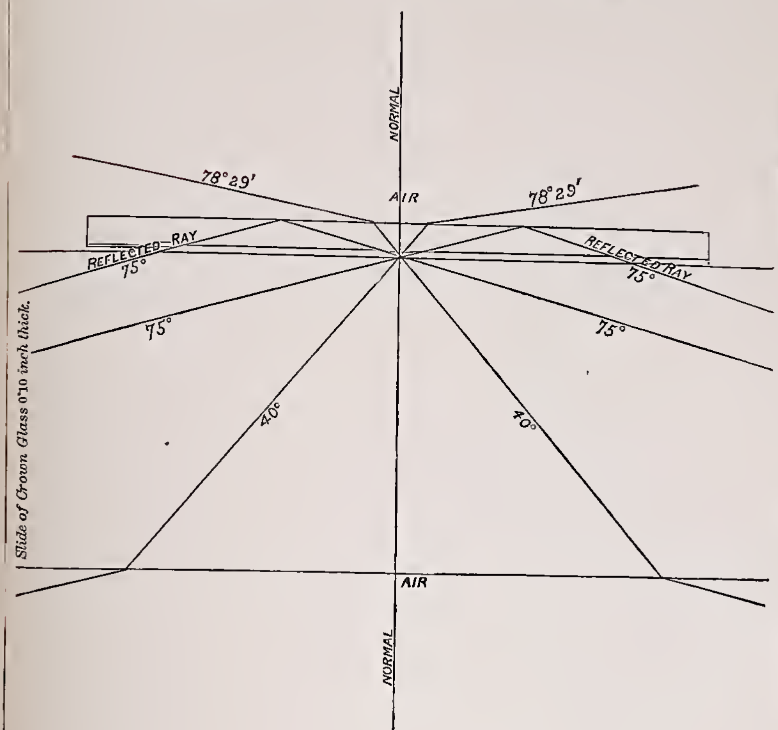

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FIGURE 14.

Balsam Mount.

Refraction of Rays from Crown Glass, index 1·525.
To Glycerine, " 1·475.

The dotted lines show the paths the rays would take if the Glycerine were replaced by Air.

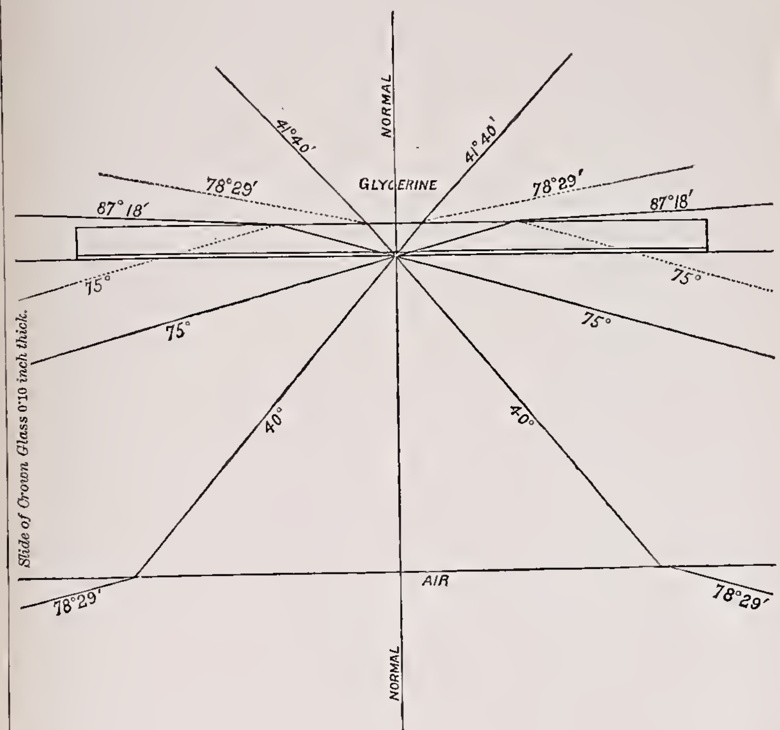

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FIGURE 15.

Balsam Mount.

Scale × 5.

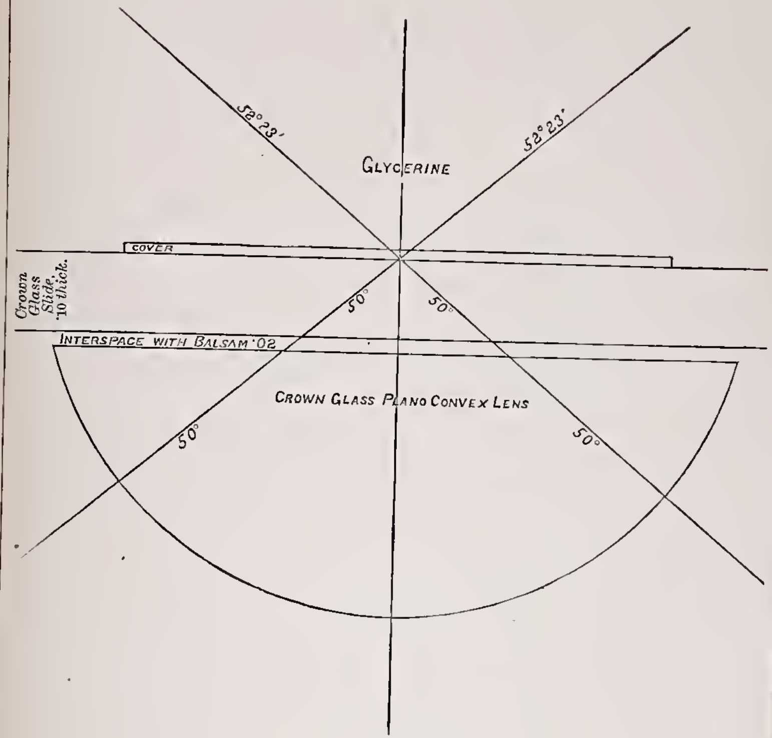

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Figure 16.

Scale × 20.

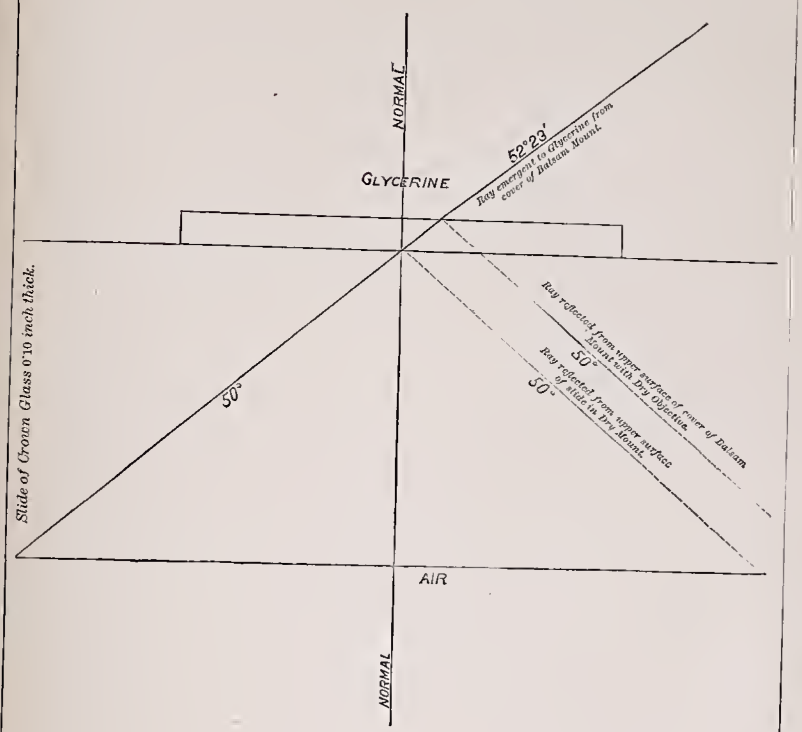

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Figure 17.

Balsam Mount.

Scale × 20.

Refraction of Rays passing from Crown Glass to Glycerine.

The dotted lines represent the course the Rays would take if the Glycerine were replaced by Air.

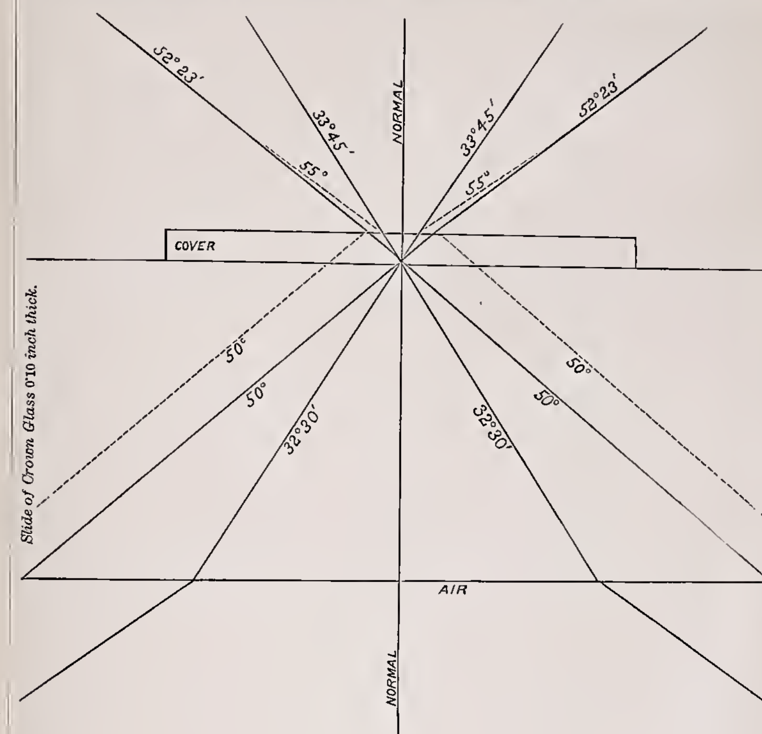

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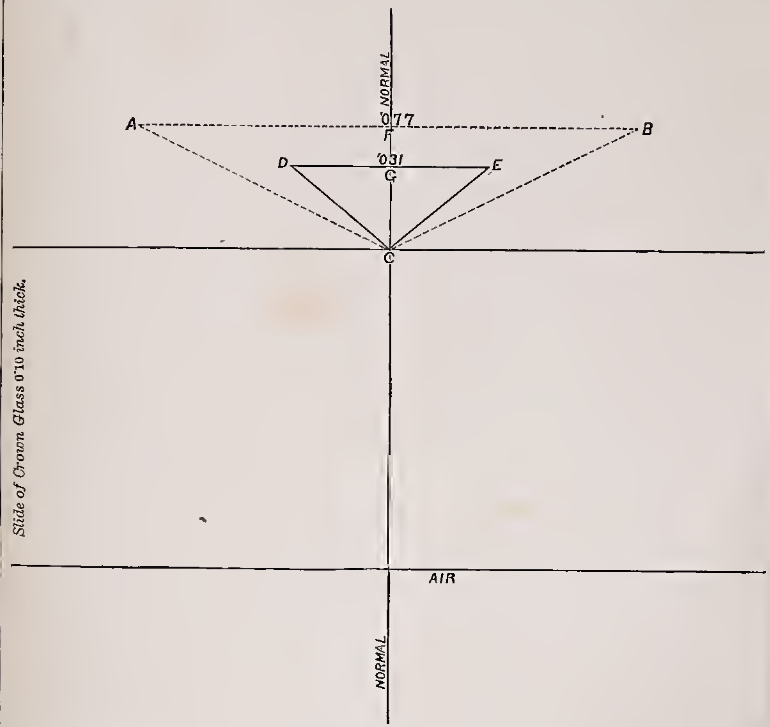
Figure 18.

Scale × 40.

Comparison of Wenham's Triangles for Tolles' Dry ¼ of 110° Air Aperture, and Tolles' Wet 1-6 of 180° Air Aperture (Glycerine Immersion).

Both in use over Balsam Mount Cover ·01 inch thick.

A. B. C. Triangle for Dry ¼.		D. E. C. Triangle for Wet 1-6.	
A. B. Clear Aperture of Front,	·077 inch.	D. E. Clear Aperture of Front,	·031 inch.
F. C. Frontal Distance,	·019 "	G. C. Frontal Distance,	·013 "
A. C. B. Angular Ap. (*a la* Wenham),	127° 28′.	D. C. E. Angular Ap. (*a la* Wenham),	100°.



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